Supplemental Document

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X-ray source translation based computed tomography (STCT): supplement

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S1. Analysis of maximum translation distance of source

Firstly, we consider a line that goes through the points (*a*, *b*). The distance from this line to the center of coordinates is *r*. we can formulate an equation of this line as

$$\begin{aligned} & y-b=k(x-a) \\ & \Rightarrow kx-y+b-ka=0 \end{aligned}, \tag{S1}$$

where *k* is the slope of this line. According to the formula for the distance between the line and a point, we have

$$\frac{|b-ka|}{\sqrt{k^2+1}}=r . \tag{S2}$$

Substituting (S1) into (S2), we can obtain that

1) In the case of $a=r$, $k=\dfrac{b^2-r^2}{2br}$;

2) In the case of $a \neq r$ and $a^2+b^2 \geq r^2$, $k=\dfrac{ab \pm \sqrt{r^2\left(a^2+b^2-r^2\right)}}{a^2-r^2}$.

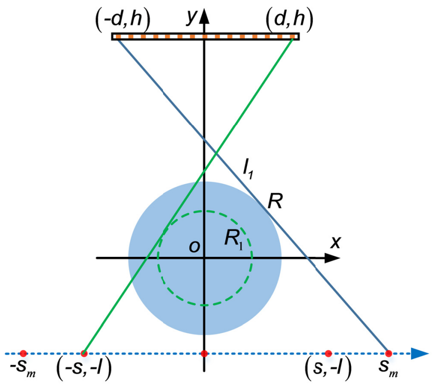

Fig. S1. The diagram of the maximum translation distance of the X-ray source.

For a detected object with radius *R*, the maximum translation range of the X-ray source is $\left[-s_m, s_m\right]$, as illustrated in Fig. S1. s_m can be calculated by finding the intersection point of the source trajectory and the line l_1 which is passing through point $(-d, h)$ and tangent to the circle $x^2+y^2=R^2$. This can be easily calculated based on the aforementioned discussion. Because the detector size is much bigger than the detected object (i.e. $d>R$) in micro CT applications, substituting $-d, h, R$ for a, b, r respectively, we can obtain the slope of l_1 as

$$k_1=\frac{-dh+\sqrt{R^2\left(d^2+h^2-R^2\right)}}{d^2-R^2}, \tag{S3}$$

Taking (S3) and $y=-l$ into (S1), we get

$$s_m=\frac{\left(d^2-R^2\right)(l+h)}{dh-R\sqrt{h^2+d^2-R^2}}-d . \tag{S4}$$

S2. Analysis of the number of translations in mSTCT

Because the angular coverage of projections from one STCT is independent of r in the case of $|r| \le R_1$, the interval angle $\Delta\theta$ is set as $2\arctan(d/h)$ to minimize data redundancy between two adjacent STCTs. Firstly, we mainly consider the angular coverage for R_1. As exhibited in Fig. S2, to guarantee the angular coverage over 360°, the number of translations T can be calculated as

$$T = \mathrm{ceil}(\psi/\Delta\theta), \tag{S5}$$

with $\psi = \pi + \Delta\theta - 2\alpha$. α is the angle between x-axis and the line perpendicular to the line l_2 as shown in Fig. S2. Since the line l_2 is tangent to the circle centered at the origin with radius R_1 while passing through the point (d,h), its slope is

$$k_3 = \frac{dh + R_1\sqrt{d^2 + h^2 - R_1^2}}{d^2 - R_1^2}. \tag{S6}$$

Therefore, α has a positive value, which can be calculated as

$$\alpha = \arctan\frac{R_1^2 - d^2}{dh + R_1\sqrt{d^2 + h^2 - R_1^2}}. \tag{S7}$$

Figure S2 displays the case of $T = 4$, each STCT has a translated angle $\theta_t = \theta_1 + (t-1)\Delta\theta$, with $\theta_1 = 0$ and $t = 1, \cdots, T$. From the distribution of projections from mSTCT, it is observed that the projection values $\bar{p}(\varphi, r)$, $\varphi \in [0, \pi]$ are available for all $|r| < R_1$ if angular coverage over 180° for $|r| = R_1$ is guaranteed. In practice, reconstructions without obvious artifacts can be obtained if T takes the nearest integer.

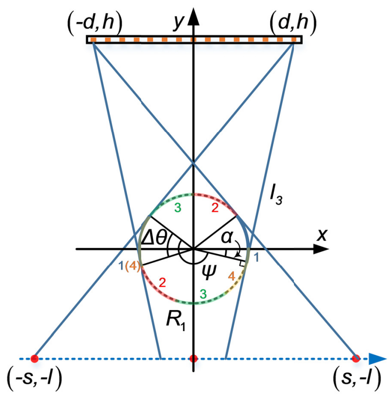

Fig. S2. The diagram of the number of translations in mSTCT. The arcs marked with 1, 2, 3, 4 represent the angular coverage for R_1 by the 1st, 2nd, 3rd, 4th translations, respectively. 1(4) indicates the angular coverage of projections can be obtained from both 1st and 4th translations. The angle of each arc is equal to $\Delta\theta$.