Supplemental Document

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Centrifugal motion of an optically levitated particle: supplement

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As shown in Fig. S1, the position vector of the levitated particle is

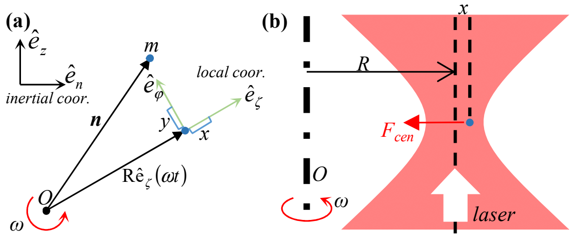

Fig. S1 Schematic illustration of centrifugal motion of a levitated particle.

$$\mathbf{n} = (R+x)\hat{e}_\zeta(\omega t) + y\hat{e}_\varphi(\omega t) \tag{S1}$$

In (S1), the transfer of the local coordinate to the inertial coordinate are as follow

$$\begin{aligned} \hat{e}_\zeta(\omega t) &= \cos(\omega t)\hat{e}_n + \sin(\omega t)\hat{e}_z \\ \hat{e}_\varphi(\omega t) &= -\sin(\omega t)\hat{e}_n + \cos(\omega t)\hat{e}_z \end{aligned} \tag{S2}$$

The net force applied on the particle is

$$m\ddot{\mathbf{n}} = -k(\mathbf{n} - R\hat{e}_\zeta) - \Gamma m(\dot{\mathbf{n}} - \dot{\mathbf{n}}_{air}) \tag{S3}$$

In which, m is the mass of the levitated particle, k is the stiffness of the optical trap, Γ is the damping coefficient, and velocity and accelerate of the particle are

$$\begin{aligned} \dot{\mathbf{n}} &= (\dot{x} - \omega y)\hat{e}_\zeta + (R\omega + x\omega + \dot{y})\hat{e}_\varphi \\ \ddot{\mathbf{n}} &= \left(\ddot{x} - 2\omega\dot{y} - \omega^2(R+x)\right)\hat{e}_\zeta + \left(\ddot{y} + 2\omega\dot{x} - \omega^2 y\right)\hat{e}_\varphi \end{aligned} \tag{S4}$$

where $2\omega\dot{x}$ and $2\omega\dot{y}$ are Coriolis accelerate, $\omega^2(R+x)$ and $\omega^2 y$ are centrifugal accelerate. $\dot{\mathbf{n}}_{air}$ is the velocity of air in a $5\times5\times1mm$ sealed chamber (Fig. 2). The velocity of air on the different position is simply approximated as

$$\dot{\mathbf{n}}_{air}(x\hat{e}_\zeta, y\hat{e}_\varphi) = \alpha(-\omega y)\hat{e}_\zeta + \beta(R\omega + x\omega)\hat{e}_\varphi \tag{S5}$$

where α and β are empirical coefficients to describe the velocity difference with chamber, $(x\hat{e}_\zeta, y\hat{e}_\varphi)$ represents the particle position. Since the particle moves relative to the chamber, the air around the particle is changed. In this case, the relative motion between the particle and the air is

$$\dot{\mathbf{n}} - \dot{\mathbf{n}}_{air} = [\dot{x} + (\alpha-1)\omega y]\hat{e}_\zeta + [\dot{y} + (1-\beta)\omega(R+x)]\hat{e}_\varphi \tag{S6}$$

Therefore, the coupled equations of motion are:

$$
\begin{aligned}
&m\left[\ddot{x}-2\omega\dot{y}-\omega^2(R+x)\right]=-kx-\Gamma m\left[\dot{x}+(\alpha-1)\omega y\right]\\
&m\left(\ddot{y}+2\omega\dot{x}-\omega^2 y\right)=-ky-\Gamma m\left[\dot{y}+(1-\beta)\omega(R+x)\right]
\end{aligned}
\tag{S7}
$$

When the turntable rotates at constant ω (steady state), the particle deviate from the center in a constant distance. Therefore, the Coriolis accelerate, velocity and accelerate in x and y can be neglected. In the steady state, the air in the chamber would follow the rotation, hence α and β can be approximated as 1. The equations of motion can be rewrite as:

$$
\begin{aligned}
&\left(\frac{k}{m}-\omega^2\right)x=\omega^2 R\\
&\left(\frac{k}{m}-\omega^2\right)y=0
\end{aligned}
\tag{S8}
$$

$\sqrt{k/m}=2\pi f=345>>\omega$ (ω<2π), f=54Hz is the transverse oscillation frequency of the trapped particle in our experiment. After ignoring the smaller value, the equilibrium position of the particle are:

$$
\begin{aligned}
&x=\frac{mR}{k}\omega^2\\
&y=0
\end{aligned}
\tag{S9}
$$