Supplemental Document

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Optical cooperative effects of multiemitters in a one-dimensional (1D) dense array: supplement

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We theoretically explore cooperative effects of equally spaced multiemitters in a 1D dense array driven by a low-intensity probe field propagating through a 1D waveguide by modeling the emitters as point-like coupled electric dipoles. We calculate the collective optical spectra of a number of 1D emitter arrays with any radiation-retention coefficient η using both exact classical-electrodynamics and mean-field-theory formalisms. We illustrate cooperative effects of lossless 1D emitter arrays with $\eta = 1$ at the emitter spacings, which are displayed by steep edges accompanied by a deep minimum and Fano resonances in the plots of transmissivities as a function of the detuning of the incident light from the emitter resonance. Numerical simulation of the full width of such optical bandgaps reveals that cooperativity between emitters is greater in a small array of size $N \leq 8$ than in a larger one of size $N > 8$. For a lossy 1D emitter array in which the radiation retention coefficient is equal to or less than 0.1 the transmissivity obtained by exact-electrodynamics scheme exhibits no bandgap structures, being in good agreement with the mean-field-theory result. We propose that a 1D multiemitter array may work as a nanoscale filter blocking transmission of light with a frequency in the range of optical bandgaps.

1. MODEL

We consider a 1D dense array of N effective two-level emitters spaced by the equal distances $a(\leq \lambda)$, residing at fixed positions $x_n(n = 1, \cdots, N)$, and driven a linearly polarized plane-wave probe field propagating through a 1D waveguide, as depicted in Fig. S1. Note the emitters are confined inside the 1D waveguide by the characteristic radial length scale denoted by ξ_ρ in Fig. S1, depicting a different system from Fig. 1. With this additional figure, we emphasize that emitters can be side coupled to the 1D waveguide. In fact, we had intended to demonstrate a sample coupled to a 1D photonic crystal waveguide [1], so called alligator. However, because of our lack of drawing skill, we have illustrate electric dipoles of emitters within a cylinder-shaped waveguide. In the limit of low light intensity the saturation of the excited state is neglected, leading to the linear regime of classical optics. There are two modes for transmission of light in the emitters. One is the usual dipolar radiation in a 3D free space. Scattered electric field from dipoles in 3D free space has been presented in [2, 3] along with the analytical expression of the steady-state electric dipole moment. We also consider the collective emission model where emitters interact through both the guided modes of the coupled 1D waveguide and 3D free-space electromagnetic field modes. We then formulate a consistent model for an 1D array of tightly confined emitters, allowing for simultaneous 1D and 3D channels for light. The collective dynamics for the optical response of the 1D multiemitter array can be described by a classical electrodynamics scheme as well as by a quantum-mechanical one.

2. TIME EVOLUTION OF EXCITATION AMPLITUDE

We here aim to derive Eq. (2) in the primary manuscript. Additionally, we validate a key idea that in the low-light-intensity limit the collective dynamics of effective two-level emitters, described in the quantum-mechanics framework, can be reduced to the classical electrodynamics equations of point-like electric dipoles.

A. One-dimensional (1D) classical electrodynamics scheme

We model the 1D channel along the lines of a 1D scalar classical electrodynamics in [4] and a more detailed analysis of the 1D light propagation in [5]. For the dynamics of dipoles we express the polarization density of emitters, which is induced by an electric field of external light driving

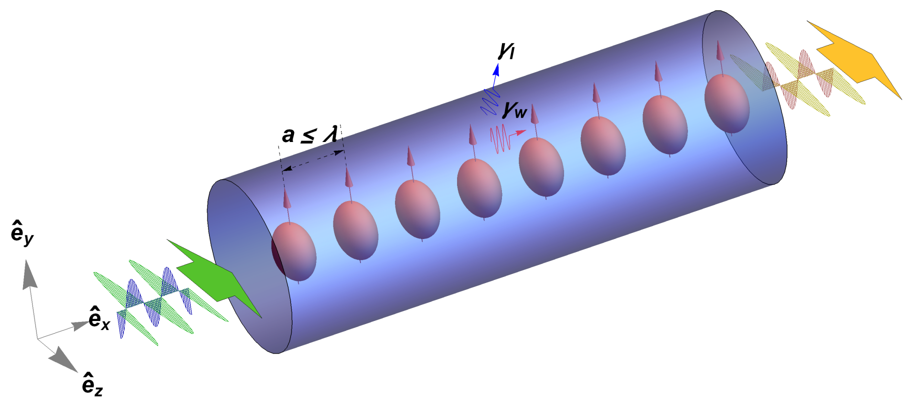

Fig. S1. A 1D array of emitters within a 1D waveguide is illustrated for the case of $N = 8$ emitters at the positions $x_n (n = 1, ..., 8)$ separated by the equal spacing of $a \leq \lambda$. Incident plane-wave probe beam with the linear polarization in the y direction propagates along the axis of the waveguide, the x direction. Emitter driven by the sum of the incident plane-wave probe beam and scattered light by the other seven emitters radiates light with rates γ_{w} into the 1D waveguide and γ_{l} into the free space. Light transmits through the 1D waveguide, inducing the identical electric dipole moments on each of the emitters in the y direction.

emitter transitions, in terms of the slowly-varying positive-frequency component without $e^{-i\omega t}$,

$$\mathbf{P}(x,t) = \sum_{n=1}^{N} \delta(x - x_n)\mathbf{d}_n(t). \tag{S1}$$

Here $\mathbf{d}_n(t)$ denotes the electric dipole moment of the emitter at fixed position x_n with fixed orientation in the y direction at time t, being abbreviation of $\mathbf{d}(x_n, t)$. For an effective two-level emitter in a $J = 0$ to $J' = 1$ transition in the absence of a magnetic field a single excited state is selected. Then the dynamics of the electric dipole moments can be represented as

$$\mathbf{d}_n(t) = \mathcal{D}\mathcal{P}_n(t)\hat{\mathbf{e}}_y, \tag{S2}$$

where $\mathcal{D}$ and $\mathcal{P}_n$ denote the dipole transition matrix element and the excitation amplitude of the emitter n, respectively. If time scale of the emitter dynamics is much longer than that for light propagation in a 1D waveguide, the polarization given in Eq. (S1) generates the dipolar electric field $\mathbf{E}^S(x_n)$ at the position x_n, scattered from all of $N-1$ emitters [6]. In terms of the monochromatic dipole field propagator G an electric field at the position x_n from a single dipole at x_l [7] reads as

$$\mathbf{E}^S(x_n) = \mathsf{G}(x_n - x_l)\mathbf{d}_l. \tag{S3}$$

All of electric dipoles in the 1D array send off electric fields, producing the dipolar electric field at the position x

$$\mathbf{E}^S(x) = \sum_{n=1}^{N} \mathsf{G}(x - x_n)\mathbf{d}_n \tag{S4}$$

summed over all dipoles. Excluding self field on an emitter by itself, an electric field of external light driving emitter transitions at the position x_n equals the sum of the electric field of the incoming plane-wave probe field of the form

$$\mathbf{E}_{\mathrm{inc}}(x) = \mathcal{E}_0\hat{\mathbf{e}}_y \exp{(ikx)} \tag{S5}$$

and the scattered field, and expressed as

$$\mathbf{E}_{\text{ext}}(x_n) = \mathbf{E}_{\text{inc}}(x_n) + \sum_{l \neq n}^{N} \mathbf{E}^S(x_l). \tag{S6}$$

After plugging the expression of a single electric dipole moment given in Eq. (S2) into Eq. (S1) describing the sample dynamics for the polarization density, we obtain below the time evolution of the excitation amplitude:

$$\frac{d\mathcal{P}_n}{dt} = (i\Delta - \gamma)\mathcal{P}_n + i\frac{\xi}{\mathcal{D}}\hat{\mathbf{e}}_y \cdot \epsilon_0 \mathbf{E}_{\text{ext}}(x_n). \tag{S7}$$

Here, $\Delta = \omega - \omega_0$ is the detuning between the incident probe field frequency and the single-emitter resonance frequency, γ the linewidth of the optical transition, and we define

$$\xi = \frac{6\pi\gamma_1}{k^3} \tag{S8}$$

for the 3D linewidth of the optical transition

$$\gamma_1 = \frac{\mathcal{D}^2 k^3}{6\pi\hbar\epsilon_0}. \tag{S9}$$

The 3D linewidth γ_1 is the radiative linewidth of a single emitter, caused by a spontaneous emission into 3D channel of light, leading to loss of light radiated by the emitter into 3D free space. Inserting Eq. (S6) with the local dipolar field $\mathbf{E}^S(x_l)$ replaced into the second term in the right-hand side of Eq. (S7) and substituting $\xi = \mathcal{D}^2/\hbar\epsilon_0$, the second term becomes

$$i\frac{\mathcal{D}^2}{\hbar\epsilon_0\mathcal{D}}\left[\epsilon_0\mathbf{E}_{\text{inc}}(x_n)\cdot\hat{\mathbf{e}}_y + \mathcal{D}\sum_{l\neq n}^{N}\mathsf{G}(x_n - x_l)\mathcal{P}_l\right] = i\kappa_0(x_n) + i\frac{\mathcal{D}^2}{\hbar\epsilon_0}\sum_{l\neq n}^{N}\mathsf{G}(x_n - x_l)\mathcal{P}_l. \tag{S10}$$

Here the definition of Rabi frequency $\kappa_0(x_n)$ given in the primary manuscript is used. Eq. (S10) indicates the excitation-amplitude dependence of the scattered dipolar electric field at the position of the emitter n. Monochromatic dipole field propagator through the 1D channel of light with an effective area $A = \pi\xi_\rho^2$ for the effective length between an emitter and the center of the waveguide is

$$\mathsf{G}(x_n - x_l) = i\frac{k}{2\epsilon_0 A}\exp\left(ik|x_n - x_l|\right) = i\frac{k}{2\pi\epsilon_0\xi_\rho^2}\exp\left(ik|x_n - x_l|\right), \tag{S11}$$

corresponding to a Green's function for the 1D Helmholtz equation [7]. Assuming a source has electric dipole moment per unit area, we obtain Eq. (S11) by integrating away the transverse components from the 3D Green's function. We mdike an assumption that the wave number of light inside the 1D channel k is the same as that in the 3D channel. Plugging $\mathsf{G}(x_n - x_l)$ into Eq. (S10), the time evolution of the excitation amplitude of emitter at position x_n is obtained below:

$$\frac{d\mathcal{P}_n}{dt} = (i\Delta - \gamma)\mathcal{P}_n + i\kappa_0(x_n) - \gamma_{\text{w}}\sum_{l\neq n}^{N}\exp\left(ik|x_n - x_l|\right)\mathcal{P}_l, \tag{S12}$$

where the single-emitter emission rate into the 1D waveguide γ_{w} is defined as

$$\gamma_{\text{w}} = \frac{\mathcal{D}^2 k}{2\pi\hbar\epsilon_0\xi_\rho^2}. \tag{S13}$$

In an ideal 1D waveguide, where all of the light emitted by the emitters are retained, the linewidth of the optical transition γ would originate from spontaneous emission into the 1D waveguide so that $\gamma = \gamma_{\text{w}}$. On the other hand, in a lossy 1D waveguide, a spontaneous emission into 3D free space should be added. Thus the linewidth of the optical transition of a single emitter is identified as the sum of the 1D and 3D linewidths, leading to the expression

$$\gamma = \gamma_{\text{w}} + \gamma_1. \tag{S14}$$

In the collective emission model under consideration, 1D waveguide-mode mediated interaction of emitters and coupling to 3D free-space electromagnetic field modes are taken into account with the radiation retention coefficient η defined as

$$\eta = \frac{\gamma_{\mathrm{W}}}{\gamma} = \frac{\gamma_{\mathrm{W}}}{\gamma_{\mathrm{W}} + \gamma_{1}}. \tag{S15}$$

Equation (S12) with $\gamma_{\mathrm{W}} = \eta\gamma$ substituted is identical with Eq. (2) in the primary manuscript. The identical equation with this has been presented for a 1D array of neutral atoms coupled to a 1D waveguide [8] and for regularly arrayed cold atoms in a planar optical lattice in a free space [6].

B. Quantum-mechanical scheme

The electric dipole matrix element $\mathbf{d}$ of an effective two-level emitter is represented by $\mathbf{d} = \mathcal{D}\hat{\mathbf{e}}_y$. With ground $|g\rangle_n$ and excited $|e\rangle_n$ states of emitter n, the emitter lowering and raising operators are $\hat{\sigma}_n^- = |g\rangle_{nn}\langle e|$ and $\hat{\sigma}_n^+ = |e\rangle_{nn}\langle g|$, respectively. In the limit of low probe light intensity, with a Gutzwiller mean-field and a rotating-wave approximations, the time evolution of the emitter coherences can obtained. The Gutzwiller mean-field approximation facilitates analysis of collective dynamics by the factorization of internal level correlations

$$\left\langle \hat{\sigma}_n^{\alpha}\hat{\sigma}_l^{\beta} \right\rangle \approx \langle \hat{\sigma}_n^{\alpha} \rangle \left\langle \hat{\sigma}_l^{\beta} \right\rangle, l \neq n. \tag{S16}$$

Emitters in our model reside at fixed positions without spatial fluctuations thus do not have correlations between the emitters after applying this approximation. In the interaction picture collective dynamics is then described by quantum master equation for the reduced density matrix ρ:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\sum_{n=1}^{N}[H_n,\rho] + i\sum_{n,l(n\neq l)}^{N}\Omega_{nl}\left[\hat{\sigma}_n^+\hat{\sigma}_l^-,\rho\right] + \sum_{nl}^{N}\gamma_{nl}\left(2\hat{\sigma}_n^-\rho\hat{\sigma}_l^+ - \hat{\sigma}_l^+\hat{\sigma}_n^-\rho - \rho\hat{\sigma}_l^+\hat{\sigma}_n^-\right). \tag{S17}$$

Here the Hamiltonian for emitter n reads

$$H_n = -\hbar\Delta\hat{\sigma}_n^{ee} - i\hbar\gamma\hat{\sigma}_n^{ee} - \mathbf{d}\cdot\mathbf{E}_{\mathrm{inc}}(x_n)\hat{\sigma}_n^+ - \mathbf{d}^*\cdot\mathbf{E}_{\mathrm{inc}}(x_n)\hat{\sigma}_n^- \tag{S18}$$

for $\hat{\sigma}_n^{ee} = \hat{\sigma}_n^+\hat{\sigma}_n^-$. Collective interaction of emitters with light has a frequency shift Ω_{nl} and a decay rate γ_{nl}, which are respectively expressed as

$$\Omega_{nl} = \frac{1}{\hbar\epsilon_0}\mathrm{Re}\left[\mathbf{d}^*\cdot\mathsf{G}(x_n - x_l)\mathbf{d}\right] \tag{S19}$$

and

$$\gamma_{nl} = \frac{1}{\hbar\epsilon_0}\mathrm{Im}\left[\mathbf{d}^*\cdot\mathsf{G}(x_n - x_l)\mathbf{d}\right]. \tag{S20}$$

In the limit of low light intensity Eq. (S16) keeps terms having at most one of either $\hat{\sigma}_n^{\pm}$ or the incident probe field amplitude. Taking the expectation values of density operators from Eq. (S17), the equations of motion is reduced to the time evolution of the coherence $\rho_{ge}^{(n)}$ of the emitter n as follows:

$$\frac{d\rho_{ge}^{(n)}}{dt} = (i\Delta - \gamma)\rho_{ge}^{(n)} + i\kappa_0(x_n) - \gamma_{\mathrm{W}}\sum_{l\neq n}^{N}\exp\left(ik|x_n - x_l|\right)\rho_{ge}^{(n)}. \tag{S21}$$

Thus the collective dynamics of emitters is reduced to that of classical coupled dipoles.

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