Supplemental Document

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Method of space object detection by wide field of view telescope based on its following error: supplement

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Supplementary S1: A clustering algorithm for associating the coordinates of suspicious objects

Suppose there are P lists of suspicious objects and their set is $\{L_1, L_2, \cdots\cdots, L_N \cdots\cdots, L_P\}$,where, $L_1 = \{(x_0, y_0)^1, (x_1, y_1)^1, \cdots, (x_i, y_i)^1, \cdots, (x_n, y_n)^1\}$ is the set of coordinates in List 1, $L_2 = \{(x_0, y_0)^2, (x_1, y_1)^2, \cdots, (x_i, y_i)^2, \cdots, (x_o, y_o)^2\}$ is the set of coordinates in List 2, $L_N = \{(x_0, y_0)^N, (x_1, y_1)^N, \cdots, (x_i, y_i)^N, \cdots, (x_p, y_p)^N\}$ is the set of coordinates in List N, and $L_P = \{(x_0, y_0)^P, (x_1, y_1)^P, \cdots, (x_i, y_i)^P, \cdots, (x_q, y_q)^P\}$ is the set of coordinates in List P.

Since the 1st frame has no corresponding list of the suspicious objects, let $L_0 = \{(x_0, y_0)^0, (x_1, y_1)^0, \cdots, (x_k, y_k)^0, \cdots, (x_r, y_r)^0\}$ be the set of coordinates of all the stars in the 1st frame.

The specific steps of the association in coordinates are as follows.

Algorithm S1 A clustering algorithm for associating the coordinates

Step 1: All the feature vectors in the set L_1 are regarded as cluster centers. For example, let the center of cluster w_0 is $z_0 = (x_0, y_0)^1$; let the center of cluster w_1 is $z_1 = (x_1, y_1)^1$; and so on; let the center of cluster w_k is $z_k = (x_i, y_i)^1$; let the center of cluster w_n is $z_n = (x_n, y_n)^1$.

Step 2: The existing cluster are $w_0, w_1, \cdots, w_k, \cdots w_n$ and the existing cluster centers are $z_0, z_1, \cdots, z_k, \cdots, z_n$. The distance is calculated between the feature vector $l_i^2 = (x_i, y_i)^2$ in the set L_2 and each cluster center $z_j (j = 0, 1, \cdots, k, \cdots, n)$. If it is true that $d_{ij} > T (j = 0, 1, \cdots\cdots, n)$, then l_i^2 is the center of the new cluster w_{n+1}, i.e. $z_{n+1} = l_i^2$. If it is true that $d_{ik} \le T$ and $d_{ik} = \min_{j=0}^{n} \left[d_{ij} \right]$, then it is judged that $l_i^2 \in w_k$ and it is set that $z_k = l_i^2$ at the same time.

Step 3: The existing cluster are $w_0, w_1, \cdots, w_k, \cdots, w_n \cdots, w_s$ and the existing cluster centers are $z_0, z_1, \cdots, z_k, \cdots, z_n, \cdots, z_s$. The distance is calculated between the feature vector $l_i^N = (x_i, y_i)^N$ in the set L_N and each cluster center $z_j (j = 0, 1, \cdots, k, \cdots, n, \cdots, s)$. If it is true that $d_{ij} > T (j = 0, 1, \cdots\cdots, s)$, then l_i^N is the center of the new cluster w_{s+1}, i.e. $z_{s+1} = l_i^N$. If it is true that $d_{ik} \le T$ and $d_{ik} = \min_{j=0}^{s} \left[d_{ij} \right]$, then it is judged that

$l_i^N \in w_k$ and it is set that $z_k = l_i^N$ at the same time. It is checked that whether all the sets are classified into cluster, if they are all classified, go to step 4; otherwise, return to step 3.

Step 4: The existing cluster are $w_0, w_1, \cdots, w_k, \cdots, w_n, \cdots, w_r, \cdots, w_t$. The number of elements in each cluster is counted and the cluster with the most elements is selected, i.e. $card(w_k) = \max\{card(w_0), \cdots, card(w_k), \cdots, card(w_t)\}$. If it is true that $card(w_k) \le P$, then the cluster w_k is the trajectory of the object in the image.

Step 5: If the feature vector $l_i^1 = (x_i, y_i)^1$ in the set L_1 belongs to the trajectory cluster w_k, then the distance d_{ij} is calculated between $l_i^1 = (x_i, y_i)^1$ and each feature vector in the set L_0. If it is true that $d_{ii} \le T$ and $d_{ii} = \min_{j=0}^{r}\left[d_{ij}\right]$, then it is judged that $l_i^0 \in w_k$. The trajectory cluster w_k at this point is the trajectory of space object containing the 1st frame.

Supplementary S2: The test for equivalent sine simulation

According to the equivalent sine simulation, when an object moves sinusoidally, the trajectory of the azimuthal axis of the telescope is

$$A = \theta_A \sin(\omega_A t), \quad \textbf{(S1)}$$

where, $\theta_A = \frac{\dot{\theta}_A^2}{\ddot{\theta}_A}$, $\omega_A = \frac{\ddot{\theta}_A}{\dot{\theta}_A}$, $\dot{\theta}_A$ is the maximum velocity of the object in azimuth and $\ddot{\theta}_A$ is the maximum acceleration in azimuth.

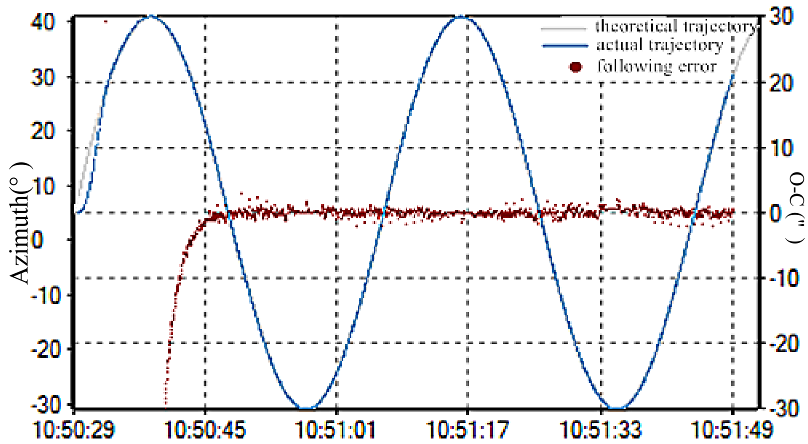
According to Table 1, the maximum velocity of the object can be set to 6 °/s and the maximum acceleration can be set to 1 °/s^2, and the trajectory of the azimuth axis of the telescope is

$$A = 36\sin(\frac{1}{6}t). \quad \textbf{(S2)}$$

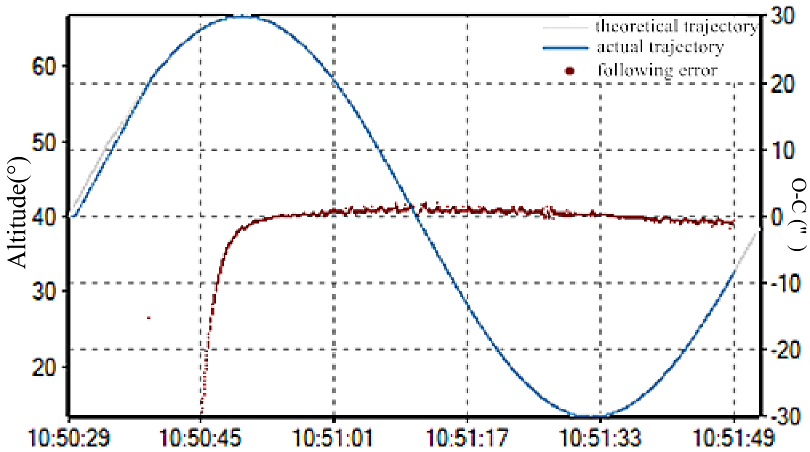
Similarly, the trajectory of the altitude axis of the telescope is

$$h = 26.67\sin(\frac{3}{40}t). \quad \textbf{(S3)}$$

The trajectory-following performance of telescope can be evaluated by the sinusoidal motion constructed with the parameters in Table 1. The test results are shown in Fig.S1. The gray curve represents the theoretical trajectory of the telescope (unit: degree), the blue curve represents the actual trajectory (unit: degree), and the red sampling points represents following error (unit: arc-second). The peaks and RMS of the following error are shown in Table S1.It can be seen that 1.2-meter survey telescope has excellent following accuracy and stability.



(a). The trajectory-following performance of azimuthal axis



(b). The trajectory-following performance of the altitude axis.

Fig. S1. The trajectory-following performance of the telescope.

Table S1. The peaks and RMS of the following error

Following error	Azimuth	Altitude
Peak(″):	$\delta_A = 2.389$	$\delta_h = 2.060$
Peak(Pixels):	$N^{\alpha}_{\max} = 1.671$	$N^{\beta}_{\max} = 1.441$
RMS(″):	$\sigma_A = 0.516$	$\sigma_h = 0.742$
RMS(Pixels):	$N^{\alpha}_{\mathrm{RMS}} = 0.362$	$N^{\beta}_{RMS} = 0.519$