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Optimal design for universal multiport interferometers: supplementary material

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This document provides supplementary information to "Optimal design for universal multiport interferometers," http://dx.doi.org/10.1364/optica.3.001460, describing (A) a method for characterizing all the elements of a realistic circuit and (B) an explicit general algorithm for implementing the decomposition described in the main text.

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A. Characterizing a realistic universal multiport interferometer

Programming a universal multiport interferometer using our procedure requires a preliminary full characterization of its beam splitters and phase shifters. This is a simple procedure, similar in spirit to that proposed by Mower *et al* [1], and only has to be done once, provided that there is no long-term drift of the optical properties of the interferometer.

At every step in the process, we choose a path through the interferometer which can be broken by setting a single beam splitter in the path to full transmission. We then input light into that path, and scan through the reflectivity of that beam splitter while monitoring the output. This allows us to characterise that beam splitter. We then set it to be fully transmissive, and move on to a different path until every beam splitter has been characterised and the interferometer implements the identity to within single-mode phase shifts.

Individual phase shifters can then be characterized by creating simple interfering paths through the interferometer, and modulating the phase shifters in those paths. Every interfering path consists of several phase shifters, but since there are many more possible interfering paths than phase shifters, the phase shifters can still be individually characterized. We note that the phase shifters at the input of the interferometer cannot be individually characterised in this way, but these are typically not relevant for most applications.

The preceding protocol assumes that the beam splitters can perfectly implement the identity. This is typically not the case for real interferometers, where small amounts of light will leak through. However, the approach proposed by Mower *et al* to overcome this problem also works for our design. This light can be isolated and removed from the characterisation process by varying the reflectivities of the beam splitters not along the path being broken, in such a way that the spurious light can be identified in the Fourier transform of the output signal.

B. General decomposition procedure

The unitary matrix decomposition procedure presented in the main text can easily be generalised to any $N \times N$ unitary matrix. Elements of $\hat{U}$ are consecutively nulled using $T_{m,n}$ or $T^{-1}_{m,n}$ matrices, which physically correspond to beam splitters in the final interferometer, in the pattern shown in figure S1.

The algorithm that implements the decomposition is the following:

Algorithm S1. Unitary Matrix Decomposition Algorithm

```
procedure DECOMPOSE(U)
    for i from 1 to N − 1 do
        if i is odd then
            for j = 0 to i − 1 do
                Find a T^{-1}_{i−j,i−j+1} matrix that nulls element (N − j, i − j) of Û
                Update Û = Û T^{-1}_{i−j,i−j+1}.
        else
            for j = 1 to i do
                Find a T_{N+j−i−1,N+j−i} matrix that nulls element (N + j − i, j) of Û
                Update Û = T_{N+j−i−1,N+j−i} Û
```

After this decomposition procedure, we obtain the following expression:

15				
7	14			
6	8	13		
2	5	9	12	
1	3	4	10	11

Fig. S1. Illustration of the order in which matrix elements of a unitary matrix $\hat{U}$ are nulled. The first element to be nulled is at the bottom left of the matrix. The following elements are then nulled in consecutive diagonals. A black element located in column i is nulled with a $T^{-1}_{i,i+1}$ matrix, and a blue element located in row i is nulled with a $T_{i-1,i}$ matrix.

$$\left(\prod_{(m,n)\in S_L} T_{m,n}\right)\hat{U}\left(\prod_{(m,n)\in S_R} T^{-1}_{m,n}\right) = D$$

where D is a diagonal matrix corresponding to single-mode phases, and S_L and S_R are the respective orderings of the (m,n) indices for the $T_{m,n}$ or $T^{-1}_{m,n}$ matrices yielded by our decomposition. This can be rewritten as:

$$\hat{U} = \left(\prod_{(m,n)\in S_L^T} T^{-1}_{m,n}\right) D \left(\prod_{(m,n)\in S_R^T} T_{m,n}\right)$$

We can then find a matrix D' and $T_{m,n}$ matrices such that the previous equation can be re-written as:

$$\hat{U} = D'\left(\prod_{(m,n)\in S} T_{m,n}\right)$$

where S is, by construction, the order of beam splitters corresponding to the desired circuit. This completes our decomposition.

REFERENCES

1. J. Mower, N. C. Harris, G. R. Steinbrecher, Y. Lahini, and D. Englund, “High-fidelity quantum state evolution in imperfect photonic integrated circuits,” Physical Review A **92**, 032322 (2015).